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The Q^2 dependence of polarized and unpolarized proton structure functions in the relativistic quark exchange framework

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Abstract. In this article we intend to discuss the evolution of polarized and unpolarized structure functions in the $(\mathrm{x}, \mathrm{Q}^2)$ plane. We analyze the proton data on the spin dependence asymmetry $\mathrm{A}_1(\mathrm{x}, \mathrm{Q}^2)$, by making the dynamical assumption that at low resolution energies, the hadrons consist only of valence quarks and the scaling violation of $\mathrm{F}_2(\mathrm{x}, \mathrm{Q}^2)$ at low x comes only from the gluons density. While the sea quark and the gluon distributions are calculated using the inverse Mellin technique and the various moments of the valence quarks, the valence quark distribution itself is obtained from the relativistic quark-exchange model. A comparison is made with the corresponding available experimental data. Finally in agreement with the data, it is demonstrated that there is no significant Q^2-dependence of asymmetry $\mathrm{A}_1(\mathrm{x}, \mathrm{Q}^2)$ for x ranging $0.014 \leq \mathrm{x} \leq 0.25$.

1 Introduction

The spin dependence structure function (SSF), $\mathrm{g}_1(\mathrm{x}, \mathrm{Q}^2)$, and the spin independence structure functions (SF), $\mathrm{F}_2(\mathrm{x}, \mathrm{Q}^2)$ or $\mathrm{F}_1(\mathrm{x}, \mathrm{Q}^2)$, resulting from deep inelastic lepton-nucleon scattering have become very important quantities in understanding of the polarized and the unpolarized quark and gluon distributions in the nucleons [1]. The above structure functions depend both on x, the fractional momentum carried by struck parton, and Q^2, the squared four-momentum transfer of the virtual photons. An experimental study of Q^2-dependence of the nucleon SSF is carried out by measuring the longitudinal asymmetry $\mathrm{A}_1(\mathrm{x}, \mathrm{Q}^2) = \mathrm{g}_1(\mathrm{x}, \mathrm{Q}^2)/\mathrm{F}_1(\mathrm{x}, \mathrm{Q}^2)$, i.e the ratio of polarized to unpolarized structure functions. The most known theoretical predictions on SSF of the nucleon were done by Bjorken [2] and Ellis and Jaffe [3] for the so called the first moment value $\Gamma_1 = \int_0^1 \mathrm{g}_1(\mathrm{x}, \mathrm{Q}^2)\, \mathrm{dx}$.

The Γ_1 value calculation requires the spin structure function $\mathrm{g}_1(\mathrm{x}, \mathrm{Q}^2)$ at the same Q^2 but in the all x ranges. To derive the moments of the structure functions from experiment a right extrapolation of the structure functions to the small x for fixed Q^2 must be done. HERRA reports on $\mathrm{F}_2(\mathrm{x}, \mathrm{Q}^2)$ reveal a rapid rise of the proton structure function $\mathrm{F}_2(\mathrm{x}, \mathrm{Q}^2)$ as x decreases below 10^{-2} [4]. According to the GLAP evolution equations [5], g_1 is expected to evolve logarithmically with Q^2. A similar Q^2 dependence has also been observed in the spin averaged structure functions $\mathrm{F}_1(\mathrm{x}, \mathrm{Q}^2)$ and $\mathrm{F}_2(\mathrm{x}, \mathrm{Q}^2)$. Although the Q^2-dependence of g_1 and F_1 are similar, but their precise behavior is sensitive to the polarized and the unpolarized quark and gluon distribution functions. At present in the most of experiments the asymmetry A_1 is taken to be Q^2-independent [4,6–8].

On the other hand, the perturbative QCD predicts that the nucleons should mainly consist of sea quarks and gluons when we work at low (high) values of x (Q^2). However, a direct measurement of the gluon density from the experiments is not possible. One should require a direct relation between $\mathrm{F}_2(\mathrm{x}, \mathrm{Q}^2)$ and the gluons distribution $\mathcal{G}(\mathrm{x}, \mathrm{Q}^2)$. In this direction, different approximations have been used in the literature to relate the Q^2-dependence of $\mathrm{F}_2(\mathrm{x}, \mathrm{Q}^2)$ to $\mathcal{G}(\mathrm{x}, \mathrm{Q}^2)$. One of the main aims of this article is to test such approximations, the details of which are defered until later on.

In order to study the Q^2-dependence of the asymmetry A_1 we proceed in the following three steps :

(i) Instead of using field operator approaches [9,10] to calculate the valence quark distributions, we use the quark-exchange formalism which was originally introduced by Hoodbhoy and Jaffe [HJ] to investigate the quark distributions in nuclear systems [11,12]. This formalism has been applied by one of the authors (MM) to light nuclei [13] and nuclear matter [14] and was recently reformulated by us to derive the spin structure function of the three-nucleon systems as well as the proton and neutron [15]. We think, this is a reasonable motivation for the initial conditions for the QCD evolution.

(*ii*) A a possible way to determine theoretically the gluon and the sea quark content of the nucleon, one can imagine the nucleon to consist essentially of valence quarks at the static point μ_0^2. The gluons are then generated through bermsstrahlung off the valence quarks in high energies and part of the gluons can materialize into the sea quarks [16].

(*iii*) Finally we calculate the spin structure function $g_1(x, Q^2)$ and the unpolarized structure functions $F_1(x, Q^2)$ and $F_2(x, Q^2)$ according to the GLAP evolution equations [5] at different values of Q^2. For the latter we also use the idea that at low x the scaling violation of structure functions are given by the gluon density alone and it does not not depend on the quark density. A comparison will also be made between the two schemes.

Thus the paper will be organized as follows: In Sect. 2 we briefly explain the quark-exchange model and we calculate the valence quark momentum distribution. In Sect. 3, the inverse Mellin transformation and the parton distributions are introduced. In Sect. 4, we discuss the Q^2-evolution of parton distributions and the unpolarized structure functions. A similar calculation will be performed for the Q^2- dependence of polarized parton distributions and the SSF. Finally, in Sect. 5 the numerical results as well as the conclusion are presented.

2 Quark-Exchange Formalism

Let us start with a brief summary of the quark-exchange formalism. We take the nucleon state to be composed of three valence quarks [15]

$$|\alpha\rangle = \mathcal{N}^{\alpha^\dagger}|0\rangle = \frac{1}{\sqrt{3!}}\mathcal{N}^{\alpha}_{\mu_1\mu_2\mu_3} q^\dagger_{\mu_1} q^\dagger_{\mu_2} q^\dagger_{\mu_3}|0\rangle \qquad (1)$$

where α (μ_i) describe the nucleon (quark) states $\{\mathbf{P}, M_S, M_T\}$ ($\{\mathbf{k}, m_s, m_t, c\}$). As usual $q^\dagger_\mu$ ($\mathcal{N}^{\alpha^\dagger}$) denote the creation operators for the quarks (nucleons) with the state index μ (α). With the convention that a repeated index means a summation as well as integration over $\mathbf{k}$, the totally antisymmetric nucleon wave function $\mathcal{N}^{\alpha}_{\mu_1\mu_2\mu_3}$ are written as,

$$\mathcal{N}^{\alpha}_{\mu_1\mu_2\mu_3} = D(\mu_1, \mu_2, \mu_3; \alpha_i) \times\delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 - \mathbf{P})\phi(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{P}) \qquad (2)$$

where $\phi(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{P})$ is the three nucleon wave function which is approximated by a Gaussian form (b $\simeq$ nucleons radius) :

$$\phi(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{P}) = \left(\frac{3b^4}{\pi^2}\right)^{\frac{3}{4}} \cdot \exp\left[-b^2\left(\frac{(k_1^2 + k_2^2 + k_3^2)}{2} + \frac{b^2P^2}{6}\right)\right] \qquad (3)$$

$D(\mu_1, \mu_2, \mu_3; \alpha_i)$ are the product of four Clebsch -Gordon coefficients, $C^{j_1j_2j}_{m_1m_2m}$, ($\epsilon_{c_1c_2c_3}$ are the color factors) which are defined as,

$$D(\mu_1, \mu_2, \mu_3; \alpha_i) = \frac{1}{\sqrt{3!}}\epsilon_{c_1c_2c_3}\frac{1}{\sqrt{2}}\sum_{s,t=0,1} C^{\frac{1}{2}s\frac{1}{2}}_{m_{s_\sigma}m_s M_{S_{\alpha_i}}} \cdot C^{\frac{1}{2}\frac{1}{2}s}_{m_{s_\mu}m_{s_\nu}m_s} C^{\frac{1}{2}t\frac{1}{2}}_{m_{t_\sigma}m_t M_{T_{\alpha_i}}} C^{\frac{1}{2}\frac{1}{2}t}_{m_{t_\mu}m_{t_\nu}m_t} \qquad (4)$$

Now, based on the nucleon creation operators, we can define the nucleus states as,

$$|\mathcal{A}_i = 3\rangle = (3!)^{-\frac{1}{2}}\chi^{\alpha_1\alpha_2\alpha_3}\mathcal{N}^{\alpha_1\dagger}\mathcal{N}^{\alpha_2\dagger}\mathcal{N}^{\alpha_3\dagger}|0\rangle \qquad (5)$$

where $\chi^{\alpha_1\alpha_2\alpha_3}$ are the complete antisymmetric nuclear wave functions (they are taken from the Faddeev calculation with the Reid soft core potential [15]) which could be interpreted as the center of mass motion of the three nucleons. Using the same definition as the one we did for the C-G coefficients in (4) i. e.

$$D(\alpha_1, \alpha_2, \alpha_3; \mathcal{A}_i) = \frac{1}{\sqrt{2}}\sum_{S,T=0,1} C^{\frac{1}{2}S\frac{1}{2}}_{M_{S_{\alpha_1}}M_S M_{S_i}} \cdot C^{\frac{1}{2}\frac{1}{2}S}_{M_{S_{\alpha_2}}M_{S_{\alpha_3}}\mathcal{M}_{\mathcal{S}}} C^{\frac{1}{2}T\frac{1}{2}}_{M_{T_{\alpha_1}}M_T M_{T_i}} \cdot C^{\frac{1}{2}\frac{1}{2}T}_{M_{T_{\alpha_2}}M_{T_{\alpha_3}}\mathcal{M}_{\mathcal{T}}} \qquad (6)$$

Then we can write the nuclear wave functions as

$$\chi^{\alpha_1\alpha_2\alpha_3} = \chi(\mathbf{P}, \mathbf{q})D(\alpha_1, \alpha_2, \alpha_3; \mathcal{A}_i) \qquad (7)$$

Relevant information comes from the momentum distribution of the constituent quarks, which can be defined for the valence quarks with fixed flavors and spin polarization, in the three nucleon system as,

$$\rho_{\bar{\mu}}(\mathbf{k}; \mathcal{A}_i) = \frac{\langle\mathcal{A}_i = 3|q^\dagger_{\bar{\mu}}q_{\bar{\mu}}|\mathcal{A}_i = 3\rangle}{\langle\mathcal{A}_i = 3|\mathcal{A}_i = 3\rangle} \qquad (8)$$

The sign bar means no summation or integration on the repeated index μ. The calculation of $\langle\mathcal{A}_i = 3|\mathcal{A}_i = 3\rangle$ would become straightforward by doing summation over $\bar{\mu}$,

$$\langle\mathcal{A}_i = 3|\mathcal{A}_i = 3\rangle = \frac{1}{9}\langle\mathcal{A}_i = 3|q^\dagger_\mu q_\mu|\mathcal{A}_i = 3\rangle = \chi^{*\alpha_1\alpha_2\alpha_3}(\delta^{\alpha_1\beta_1}\delta^{\alpha_2\beta_2}\delta^{\alpha_3\beta_3} - \mathcal{E}^{\alpha_1\alpha_2\alpha_3,\beta_1\beta_2\beta_3}_{\mu\mu})\chi^{\beta_1\beta_2\beta_3}$$

where

$$\mathcal{E}^{\alpha_1\alpha_2\alpha_3,\beta_1\beta_2\beta_3}_{\mu\mu} = \mathcal{N}^{\alpha_2}_{\mu_1\mu_2\mu_3}\mathcal{N}^{\beta_2}_{\mu_2\mu_3\rho_1} \cdot\mathcal{N}^{\alpha_3}_{\rho_1\rho_2\rho_3}\mathcal{N}^{\beta_3}_{\mu_1\rho_2\rho_3}\delta^{\alpha_1\beta_1} \qquad (9)$$

After performing some algebra, one can drive the following equation for the expectation value of $q^\dagger q$,

$$\langle\mathcal{A}_i = 3|q^\dagger_{\bar{\mu}}q_{\bar{\mu}}|\mathcal{A}_i = 3\rangle = 9\chi^{*\alpha_1\alpha_2\alpha_3}(\mathcal{U}^{\alpha_1\alpha_2\alpha_3,\beta_1\beta_2\beta_3}_{\bar{\mu}\bar{\mu}} - \mathcal{V}^{\alpha_1\alpha_2\alpha_3,\beta_1\beta_2\beta_3}_{\bar{\mu}\bar{\mu}})\chi^{\beta_1\beta_2\beta_3}$$

where

$$\mathcal{U}^{\alpha_1\alpha_2\alpha_3,\beta_1\beta_2\beta_3}_{\bar{\mu}\bar{\mu}} = \mathcal{N}^{\alpha_1}_{\bar{\mu}\sigma_2\sigma_3}\mathcal{N}^{\beta_1}_{\bar{\mu}\sigma_2\sigma_3}\delta^{\alpha_2\beta_2}\delta^{\alpha_3\beta_3} \tag{10}$$

and

$$\begin{aligned}\mathcal{V}^{\alpha_1\alpha_2\alpha_3,\beta_1\beta_2\beta_3}_{\bar{\mu}\bar{\mu}} &= 3\mathcal{N}^{\alpha_1}_{\bar{\mu}\sigma_2\sigma_3}\mathcal{N}^{\beta_1}_{\bar{\mu}\sigma_2\sigma_3}\mathcal{N}^{\alpha_2}_{\mu_1\mu_2\mu_3}\mathcal{N}^{\beta_2}_{\rho_1\mu_2\mu_3}\\ &\quad\cdot\mathcal{N}^{\alpha_3}_{\rho_1\rho_2\rho_3}\mathcal{N}^{\beta_3}_{\mu_1\rho_2\rho_3}\\ &\quad+4\mathcal{N}^{\alpha_2}_{\bar{\mu}\mu_1\mu_2}\mathcal{N}^{\beta_2}_{\bar{\mu}\mu_2\rho_1}\mathcal{N}^{\alpha_3}_{\rho_1\rho_2\rho_3}\mathcal{N}^{\beta_3}_{\mu_1\rho_2\rho_3}\delta^{\alpha_1\beta_1}\\ &\quad+2\mathcal{N}^{\alpha_2}_{\mu_1\mu_2\mu_3}\mathcal{N}^{\beta_2}_{\bar{\mu}\mu_2\mu_3}\mathcal{N}^{\alpha_3}_{\bar{\mu}\rho_2\rho_3}\mathcal{N}^{\beta_2}_{\mu_1\rho_2\rho_3}\delta^{\alpha_1\beta_1}\end{aligned} \tag{11}$$

Assuming the nucleus to be in the rest frame and defining the Fourier transform of $\chi(\mathbf{P},\mathbf{q})$, we can calculate the expectation values of different terms in the (10) and (11)

$$\begin{aligned}&\chi^{*\alpha_1\alpha_2\alpha_3}\mathcal{N}^{\alpha_1}_{\bar{\mu}\sigma_2\sigma_3}\mathcal{N}^{\beta_1}_{\bar{\mu}\sigma_2\sigma_3}\delta^{\alpha_2\beta_2}\delta^{\alpha_3\beta_3}\chi^{\beta_1\beta_2\beta_3}\\ &\quad=\left(\frac{3\mathrm{b}^2}{2\pi^2}\right)^{\frac{3}{2}}\exp[-\frac{3}{2}\mathrm{b}^2\mathbf{k}^2]\mathrm{D}(\bar{\mu},\sigma_2,\sigma_3;\alpha_1)\\ &\qquad\cdot\mathrm{D}(\bar{\mu},\sigma_2,\sigma_2,\sigma_3;\beta_1)\mathrm{D}(\alpha_1,\alpha_2,\alpha_3;\mathcal{A}_{\mathrm{i}})\\ &\qquad\cdot\mathrm{D}(\beta_1,\beta_2,\beta_3;\mathcal{A}_{\mathrm{i}})\delta^{\alpha_2\beta_2}\delta^{\alpha_3\beta_3}\end{aligned} \tag{12}$$

$$\begin{aligned}&\chi^{*\alpha_1\alpha_2\alpha_3}\mathcal{N}^{\alpha_1}_{\bar{\mu}\sigma_2\sigma_3}\mathcal{N}^{\beta_1}_{\bar{\mu}\sigma_2\sigma_3}\mathcal{N}^{\alpha_2}_{\mu_1\mu_2\mu_3}\mathcal{N}^{\beta_2}_{\rho_1\mu_2\mu_3}\mathcal{N}^{\alpha_3}_{\rho_1\rho_2\rho_3}\\ &\cdot\mathcal{N}^{\beta_3}_{\mu_1\rho_2\rho_3}\chi^{\beta_1\beta_2\beta_3}\\ &=\mathrm{I}\left(\frac{27\mathrm{b}^2}{8\pi^2}\right)^{\frac{3}{2}}\exp[-\frac{3}{2}\mathrm{b}^2\mathbf{k}^2]\mathrm{D}(\bar{\mu},\sigma_2,\sigma_3;\alpha_1)\\ &\quad\cdot\mathrm{D}(\bar{\mu},\sigma_2,\sigma_3;\beta_1)\mathrm{D}(\mu_1,\mu_2,\mu_3;\alpha_2)\\ &\quad\cdot\mathrm{D}(\rho_1,\mu_2,\mu_3;\beta_2)\mathrm{D}(\rho_1,\rho_2,\rho_3;\alpha_3)\\ &\quad\cdot\mathrm{D}(\mu_1,\rho_2,\rho_3;\beta_3)\mathrm{D}(\alpha_1,\alpha_2,\alpha_3;\mathcal{A}_{\mathrm{i}})\\ &\quad\cdot\mathrm{D}(\beta_1,\beta_2,\beta_3;\mathcal{A}_{\mathrm{i}})\end{aligned} \tag{13}$$

$$\begin{aligned}&\chi^{*\alpha_1\alpha_2\alpha_3}\mathcal{N}^{\alpha_2}_{\bar{\mu}\mu_1\mu_2}\mathcal{N}^{\beta_2}_{\bar{\mu}\mu_2\rho_1}\mathcal{N}^{\alpha_3}_{\rho_1\rho_2\rho_3}\mathcal{N}^{\beta_3}_{\mu_1\rho_2\rho_3}\delta^{\alpha_1\beta_1}\chi^{\beta_1\beta_2\beta_3}\\ &=\mathrm{I}\left(\frac{27\mathrm{b}^2}{7\pi^2}\right)^{\frac{3}{2}}\exp[-\frac{12}{7}\mathrm{b}^2\mathbf{k}^2]\mathrm{D}(\mu_1,\mu_2,\bar{\mu};\alpha_2)\\ &\quad\cdot\mathrm{D}(\rho_1,\mu_2,\bar{\mu};\beta_2)\mathrm{D}(\rho_1,\rho_2,\rho_3;\alpha_3)\\ &\quad\cdot\mathrm{D}(\mu_1,\rho_2,\rho_3;\beta_3)\mathrm{D}(\alpha_1,\alpha_2,\alpha_3;\mathcal{A}_{\mathrm{i}})\\ &\quad\cdot\mathrm{D}(\beta_1,\beta_2,\beta_3;\mathcal{A}_{\mathrm{i}})\delta^{\alpha_1\beta_1}\end{aligned} \tag{14}$$

$$\begin{aligned}&\chi^{*\alpha_1\alpha_2\alpha_3}\mathcal{N}^{\alpha_2}_{\mu_1\mu_2\mu_3}\mathcal{N}^{\beta_2}_{\bar{\mu}\mu_2\mu_3}\mathcal{N}^{\alpha_3}_{\bar{\mu}\rho_2\rho_3}\mathcal{N}^{\beta_2}_{\mu_1\rho_2\rho_3\delta^{\alpha}_1\beta_1}\chi^{\beta_1\beta_2\beta_3}\\ &=\mathrm{I}\left(\frac{27\mathrm{b}^2}{4\pi^2}\right)^{\frac{3}{2}}\exp[-3\mathrm{b}^2\mathbf{k}^2]\mathrm{D}(\mu_1,\mu_2,\mu_3;\alpha_2)\\ &\quad\cdot\mathrm{D}(\mu_2,\mu_3,\bar{\mu};\beta_2)\mathrm{D}(\bar{\mu},\rho_2,\rho_3;\alpha_3)\\ &\quad\cdot\mathrm{D}(\mu_1,\rho_2,\rho_3;\beta_3)\mathrm{D}(\alpha_1,\alpha_2,\alpha_3;\mathcal{A}_{\mathrm{i}})\\ &\quad\cdot\mathrm{D}(\beta_1,\beta_2,\beta_3;\mathcal{A}_{\mathrm{i}})\delta^{\alpha_1\beta_1}\end{aligned} \tag{15}$$

$$\begin{aligned}&\chi^{*\alpha_1\alpha_2\alpha_3}\delta^{\alpha_1\beta_1}\mathcal{N}^{\alpha_2}_{\mu_1\mu_2\mu_3}\mathcal{N}^{\beta_2}_{\mu_2\mu_3\rho_1}\mathcal{N}^{\alpha_3}_{\rho_1\rho_2\rho_3}\mathcal{N}^{\beta_3}_{\mu_1\rho_2\rho_3\delta^{\alpha}_1\beta_1}\chi^{\beta_1\beta_2\beta_3}\\ &=\mathrm{I}\left(\frac{3}{2}\right)^{3}\mathrm{D}(\mu_1,\mu_2,\mu_3;\alpha_2)\mathrm{D}(\rho_1,\mu_2,\mu_3;\beta_2)\\ &\quad\cdot\mathrm{D}(\rho_1,\rho_2,\rho_3;\alpha_3)\mathrm{D}(\mu_1,\rho_2,\rho_3;\beta_3)\\ &\quad\cdot\mathrm{D}(\alpha_1,\alpha_2,\alpha_3;\mathcal{A}_{\mathrm{i}})\mathrm{D}(\beta_1,\beta_2,\beta_3;\mathcal{A}_{\mathrm{i}})\delta^{\alpha_1\beta_1}\end{aligned} \tag{16}$$

where

$$\begin{aligned}\mathrm{I} = 8\pi^2\int_0^\infty \mathrm{x}^2\ \mathrm{dx}\int_0^\infty \mathrm{y}^2\ \mathrm{dy}\int_{-1}^{1}\ \mathrm{d}(\cos\theta)\\ \cdot\exp[-\frac{3\mathrm{x}^2}{4\mathrm{b}^2}]|\chi(\mathrm{x},\mathrm{y},\cos\theta)|^2\end{aligned} \tag{17}$$

The above equations, i.e. (12)-(16), have been derived using the same approximation as used by HJ [11,12] and other authors [13], specially a leading order expansion for $\chi(\mathbf{P},\mathbf{q})$ [15].

3 Inverse Mellin transformation and parton distribution functions

The most natural possibility is, to consider the nucleon as consisting entirely of valence quarks at the static point μ_0^2, and generating the gluon and sea quark distributions purely radiatively at $\mathrm{Q}^2 > \mu_0^2$ [16]. We have two essential cases : (A) Unpolarized and (B) Polarized.

3.1 Unpolarized Parton Distributions

It is well known that the Q^2 dependence implied by QCD could be simply expressed in terms of the parton density moments. One therefore can write in the N-moment space,

$$M_{\mathcal{P}}(n,Q^2) = \int_0^1 x^{n-1}\mathcal{P}(x,Q^2)\ dx \tag{18}$$

where $\mathcal{P} = \mathrm{q}^{\mathrm{v}}, \bar{\mathrm{q}}, \mathrm{G}$ and $\mathrm{M}_{\mathcal{P}}(\mathrm{n},\mathrm{Q}^2)$ is the Mellin transform of the parton distribution, $\mathcal{P}(\mathrm{x},\mathrm{Q}^2)$. Here we assume an SU(3) flavor-symmetric sea quark distributions $\bar{\mathrm{q}} = \bar{\mathrm{u}} = \bar{\mathrm{d}} = \bar{\mathrm{s}} = \mathrm{s}$. In addition we consider the sea quark and the gluon contributions to vanish in the static point μ_0^2, i.e.,

$$G(x,\mu_0^2) = 0 \qquad \bar{q}(x,\mu_0^2) = 0 \tag{19}$$

The evolution procedure yields the gluon and the sea quark distributions.

$$\begin{aligned}\sum_{\mathrm{q=u,d}}\mathrm{M}^{\mathrm{v}}_{\mathrm{q}}(\mathrm{n},\mathrm{Q}^2) &= \Big\{\sum_{\mathrm{q=u,d}}\mathrm{M}^{\mathrm{v}}_{\mathrm{q}}(\mathrm{n},\mu_0^2)\Big\}\mathrm{L}^{-\mathrm{a}^{\mathrm{n}}_{\mathrm{NS}}}\\ \mathrm{M}_{\bar{\mathrm{q}}}(\mathrm{n},\mathrm{Q}^2) &= \frac{1}{6}\Big\{\sum_{\mathrm{q=u,d}}\mathrm{M}^{\mathrm{v}}_{\mathrm{q}}(\mathrm{n},\mu_0^2)\Big\}\\ &\quad\cdot\ (\alpha^{\mathrm{n}}\mathrm{L}^{-\mathrm{a}^{\mathrm{n}}_{-}} + (1-\alpha_{\mathrm{n}})\mathrm{L}^{-\mathrm{a}^{\mathrm{n}}_{+}} - \mathrm{L}^{-\mathrm{a}^{\mathrm{n}}_{\mathrm{NS}}})\\ \mathrm{M}_{\mathrm{G}}(\mathrm{n},\mathrm{Q}^2) &= \Big\{\sum_{\mathrm{q=u,d}}\mathrm{M}^{\mathrm{v}}_{\mathrm{q}}(\mathrm{n},\mu_0^2)\Big\}\frac{\alpha^{\mathrm{n}}(1-\alpha^{\mathrm{n}})}{\beta^{\mathrm{n}}}\\ &\quad\cdot\ (\mathrm{L}^{-\mathrm{a}^{\mathrm{n}}_{-}} - \mathrm{L}^{-\mathrm{a}^{\mathrm{n}}_{+}})\end{aligned} \tag{20}$$

where $a_i^n = -2\frac{P_i^n}{\beta_0}$ and β_0, α^n and β^n are written as following,

$$\begin{aligned}\beta_0 &= \left(11 - \frac{2}{3}N_f\right)\\ \alpha^n &= \frac{P^n_{qq} - P^n_{+}}{P^n_{-} - P^n_{+}}\\ \beta^n &= \frac{P^n_{qg}}{P^n_{-} - P^n_{+}}\end{aligned} \tag{21}$$

N_f is the number of active quark flavors (in our case $N_f = 2, 3$ and 4) and

$$\begin{aligned}P^n_{\pm} &= \frac{1}{2}[P^n_{qq} + P^n_{gg} \pm \sqrt{(P^n_{gg} - P^n_{qq})^2 + 4P^n_{qg}P^n_{gq}}]\\ P^n_{NS} &= P^n_{qq} = \frac{4}{3}[\frac{3}{2} + \frac{1}{n(n+1)} - 2S_1(n)]\\ P^n_{qg} &= \frac{(1}{2)}\frac{(n^2+n+2)}{n(n+1)(n+2)}\\ P^n_{gq} &= \frac{4}{3}\frac{(N^2+n+2)}{n(n^2-1)}\\ P^n_{gg} &= 3[\frac{11}{6} - \frac{N_f}{9} + \frac{2}{n(n-1)}\frac{2}{(n+1)(n+2)} - 2S_1(n)]\end{aligned} \tag{22}$$

with

$$\begin{aligned}S_1(n) &= \psi(n+1) + \gamma_E,\\ \psi(n) &= \frac{d}{dn}\ln\Gamma(n),\\ \gamma_E &= 0.5772\end{aligned} \tag{23}$$

where γ_E is the Euler's constant [17].

The quantity L, the coupling ratio, is defined as (Λ^2 is the QCD cut off parameter),

$$L = \frac{\alpha_s(\mu_0^2)}{\alpha_s(Q^2)} = \frac{\ln(\frac{Q^2}{\Lambda^2})}{\ln(\frac{\mu_0^2}{\Lambda^2})} \tag{24}$$

and can be related to the second moment of the nucleon (i.e. the proton and the neutron on average) structure function [18,19]

$$\begin{aligned}M^N(2, Q^2) &= \int_0^1 F_2^N(x, Q^2)\ dx\\ &= \frac{2}{9}\left[\frac{9}{25} + \frac{16}{25}L^{-\frac{50}{81}}\right] + \frac{1}{18}L^{-\frac{32}{81}}\end{aligned} \tag{25}$$

Experimentally [20] $M^N(2, Q^2) = 0.127$ at $Q^2 = 15$ GeV^2. So one can calculate L at the other Q^2 values.

All of the above solutions in the N-moment space can be inverted into the x-space by applying the inverse Mellin ($\mathcal{M}^{-1}$) transformation of (19) [21]

$$\begin{aligned}\mathcal{P}(x, Q^2) &= \mathcal{M}^{-1}[M_{\mathcal{P}}(n, Q^2)]\\ &= \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} x^{-n}M_{\mathcal{P}}(n, Q^2)\ dn\end{aligned} \tag{26}$$

3.2 Polarized Parton Distributions

Like the unpolarized case, the Q^2-dependence can be calculated from the parton moments, which are defined as:

$$\Delta M_{\mathcal{P}}(n, Q^2) = \int_0^1 x^{n-1}\Delta\mathcal{P}(x, Q^2)\ dx \tag{27}$$

where $\mathcal{P} = q^v, \bar{q}, G$ and $\Delta M_{\mathcal{P}}(n, Q^2)$ are the Mellin transforms of the polarized parton distributions, $\Delta\mathcal{P}(x, Q^2)$. Again we assume that the SU(3) flavor-symmetry is valid, and that the sea quark and the gluon contributions are no longer exist at μ_0^2.

As in the unpolarized case, the polarized parton moments obey a similar set of equations, i.e. (19) for the Q^2-evolution, but with the following modification in the splitting function and some other parameters, as

$$\begin{aligned}\Delta P^n_{\pm} &= \frac{1}{2}\Big[\Delta P^n_{qq} + \Delta P^n_{gg}\\ &\qquad \pm\sqrt{(\Delta P^n_{gg} - \Delta P^n_{qq})^2 + 4\Delta P^n_{qg}\Delta P^n_{gq}}\Big]\\ \Delta P^n_{NS} &= \Delta P^n_{qq} = \frac{4}{3}[\frac{3}{2} + \frac{1}{n(n+1)} - 2S_1(n)]\\ \Delta P^n_{qg} &= N_f\frac{(n-1)}{n(n+1)}\\ \Delta P^n_{gq} &= \frac{4}{3}\frac{(n+2)}{n(n+1)}\\ \Delta P^n_{gg} &= 3[\frac{11}{6} - \frac{N_f}{9} + \frac{4}{n(n+1)} - 2S_1(n)]\end{aligned} \tag{28}$$

4 Q^2 dependence of parton distributions

By considering the relativistic corrections, the valence parton distribution at each Q^2 can be related to momentum distribution for each flavor according to the following equation [15,22]

$$\begin{aligned}q^v(x, Q^2; \mathcal{A}_i) &= \frac{1}{(1-x)^2}\\ &\quad\cdot\int \rho_q(\mathbf{k}; \mathcal{A}_i)\delta\left(\frac{x}{(1-x)} - \frac{k_+}{M}\right) d\mathbf{k}\end{aligned} \tag{29}$$

After doing the angular integration, we get,

$$q^v(x, Q^2; \mathcal{A}_i) = \frac{2\pi M}{(1-x)^2}\int_{k_{min}}^{\infty} \rho_q(\mathbf{k}; \mathcal{A}_i)k\ dk \tag{30}$$

with

$$k_{min}(x) = \frac{(\frac{xM}{1-x} + \epsilon_0)^2 - m^2}{2(\frac{xM}{1-x} + \epsilon_0)} \tag{31}$$

where m (M) is the quark (nucleon) mass , k_+ is the light-cone momentum of initial quark and ϵ_0 is the quark binding energy. For each Q^2-value, we are able to calculate the corresponding values of m and ϵ_0. The valence quark distributions of a bound nucleon can be derived from the free nucleon valence quark distribution function by using the convolution approximation,

$$q^v(x,Q^2;\mathcal{A}_i)=\sum_N\int q^v(\frac{x}{y_{\mathcal{A}_i}},Q^2;N)f_{N/\mathcal{A}_i}(y_{\mathcal{A}_i})\;dy_{\mathcal{A}_i} \tag{32}$$

where $\mathrm{f}_{N/\mathcal{A}_i}(\mathrm{y}_{\mathcal{A}_i})$ is the nucleon momentum distribution in the nucleus. By taking into account the fact that $\mathrm{f}_{N/\mathcal{A}_i}(\mathrm{y}_{\mathcal{A}_i})$ is large only around $\frac{\mathrm{x}}{\langle \mathrm{y}_{\mathcal{A}_i}\rangle}$ we can write [15]

$$q^v(\frac{x}{\langle y_{\mathcal{A}_i}\rangle},Q^2;N)=q^v(x,Q^2;\mathcal{A}_i) \tag{33}$$

with $\langle \mathrm{y}_{\mathcal{A}_i}\rangle=1+\frac{\bar{\epsilon}}{\mathrm{M}}$ and $\bar{\epsilon}$ being the average removal energy of the nucleon. In general, we know that, this is not a good approximation, but having said that , there is not much difference between the structure functions of ^{3}He (^{3}H) (obviously per nucleon) and the neutron (proton). So even without using such an approximation one can also takes ^{3}He (^{3}H) as the SSF of neutrons and (protons) [15].

A typical ansatz for the parton distribution is usually parameterized as [23]

$$x\;\mathcal{P}(x,Q^2)=A_{\mathcal{P}}\eta_{\mathcal{P}}x^{a_{\mathcal{P}}}(1-x)^{b_{\mathcal{P}}}(1+\gamma_{\mathcal{P}}x+\varrho_{\mathcal{P}}x^{\frac{1}{2}}) \tag{34}$$

where $\mathrm{A}_{\mathcal{P}}$ the normalization factor is given by,

$$\mathrm{A}_{\mathcal{P}}^{-1}=\left(1+\gamma_{\mathcal{P}}\frac{\mathrm{a}_{\mathcal{P}}}{\mathrm{a}_{\mathcal{P}}+\mathrm{b}_{\mathcal{P}}+1}\beta(\mathrm{a}_{\mathcal{P}}+\mathrm{b}_{\mathcal{P}}-1,\mathrm{b}_{\mathcal{P}}+1)\right)+\varrho_{\mathcal{P}}\beta(\mathrm{a}_{\mathcal{P}}+\mathrm{b}_{\mathcal{P}}-\frac{1}{2},\mathrm{b}_{\mathcal{P}}+1) \tag{35}$$

The above parameters are calculated by using the quark-exchange model and the inverse Mellin transformation method.

The nucleon structure function $\mathrm{F}_2(\mathrm{x},\mathrm{Q}^2)$ at given Q^2 can be expressed in terms of the various parton distributions (the unpolarized valence distributions are obtained by making a spin-average on the polarized valence quark distributions) i.e.,

$$F_2(x,Q^2)=x\sum_{q,\bar{q}}e_q^2[q(x,Q^2)+\bar{q}(x,Q^2)] \tag{36}$$

On the basis of GLAP equations it is well known that at low x the dominant source for the $\mathrm{F}_2(\mathrm{x},\mathrm{Q}^2)$ scaling violations is the transition of gluons into quark-antiquark pairs [24–26]. So it is possible to relate the gluon density directly to the Q^2 dependence of the structure function $\mathrm{F}_2(\mathrm{x},\mathrm{Q}^2)$ i.e.,

$$\frac{\partial F_2(x,Q^2)}{\partial \ln Q^2}=2\sum_q e_q^2\frac{\alpha_s}{4\pi}\int_0^1\mathcal{G}(\frac{x}{z,Q^2})P_{qg}(z)\;dz \tag{37}$$

where $\mathcal{G}(\mathrm{x},\mathrm{Q}^2)=\mathrm{x}\;\mathrm{G}(\mathrm{x},\mathrm{Q}^2)$ and P_{qg} is the quark -gluon splitting function i.e. [24],

$$P_{qg}=(1-z)^2+z^2$$

Several approximate methods have been used to deconvolute the gluon density directly from $\mathrm{F}_2(\mathrm{x},\mathrm{Q}^2)$ [24–26]. All these methods are based on the simplification of convolution given in the equation (37) by making an expansion for the gluon density. The result is the gluon density $\mathcal{G}$(k.x) (which is proportional to the derivative of $\mathrm{F}_2(\mathrm{x},\mathrm{Q}^2)$) where the constant k is associated with the point which we expand the gluon density around it. The expansion of the gluon density about an arbitrary point $\mathrm{z}=\alpha$ up to the first derivative (which is valid in the limit of $\mathrm{x}\to 0$) relates the evolution of SF to the gluon density according to [26],

$$\frac{\partial F_2(x,Q^2)}{\partial \ln Q^2}=2\sum_q e_q^2\frac{\alpha_s}{4\pi}\frac{2}{3}\mathcal{G}(\frac{x}{1-\alpha}(\frac{3}{2}-\alpha),Q^2) \tag{38}$$

To evaluate $\mathrm{F}_2(\mathrm{x},\mathrm{Q}^2)$ from $\mathcal{G}(\mathrm{x},\mathrm{Q}^2)$, we must parameterize the calculated gluon distribution at different Q^2 in terms of gluon density which has been already calculated at $\mathrm{Q}_0^2=4\mathrm{GeV}^2$ in Sect. 3.1 i.e.,

$$\begin{aligned}\mathcal{G}(\mathrm{x},\mathrm{Q}^2)&=\mathcal{T}(\mathrm{x};\mathrm{Q}^2,\mathrm{Q}_0^2)\mathcal{G}(\mathrm{x},\mathrm{Q}_0^2)\\ \mathcal{T}(\mathrm{x};\mathrm{Q}^2,\mathrm{Q}_0^2)&=\mathcal{T}_1(\mathrm{x};\mathrm{Q}_0^2)+\mathcal{T}_2(\mathrm{x};\mathrm{Q}_0^2)\ln\mathrm{Q}^2\end{aligned} \tag{39}$$

where $\mathcal{T}_1(\mathrm{x};\mathrm{Q}_0^2)$ and $\mathcal{T}_2(\mathrm{x};\mathrm{Q}_0^2)$ are defined as following

$$\begin{aligned}\mathcal{T}_1(\mathrm{x};\mathrm{Q}_0^2)&=0.8354+.2064\;\mathrm{x}^{0.5}\\ \mathcal{T}_2(\mathrm{x};\mathrm{Q}_0^2)&=0.1297-0.1245\;\mathrm{x}^{0.5}\end{aligned} \tag{40}$$

For the polarized valence parton distributions at different Q^2, we do not need the above prescription. After implementing the formalism of Sect. 4 and [15], according to the leading order QCD parton model, the SSF of the nucleon at different values of Q^2 with inclusion of the higher order corrections can be expressed as [27],

$$g_1(x;Q^2)=\frac{1}{2}\sum_{q=u,d,s}e_q^2\{\Delta q^v(x,Q^2)+\Delta\bar{q}(x,Q^2)\} \tag{41}$$

5 Results and discussions

In Figs. 1 to 6 we have plotted the unpolarized proton structure function $\mathrm{F}_2(\mathrm{x},\mathrm{Q}^2)$ versus Q^2 for different values of x i.e., 0.0001, 0.0004, 0.001, 0.0025, 0.004 and 0.0063. The data are taken from [4].

In each of these figures the dashed curve is evaluated by using (36). So, first, the valence unpolarized parton distribution is calculated from Sect. 2 at the static point μ_0^2 and second, the evolution equation in Sect. 3.1 is solved to obtain the corresponding valence and sea-quark contributions at some Q^2 value. The dotted and heavy full curves represent also $\mathrm{F}_2(\mathrm{x},\mathrm{Q}^2)$, the proton SF, but evaluating the sea-quark contribution from (37) with 3 and 4 flavor (note that only the factor e_q^2 will be changed in (37)) respectively. By comparing these figures, we can conclude that the approximations leading to (37) is not very good. Since the full and dotted curves have different normalization and slop respect to the dashed curves. So one can argue that the analytical approximation to the GLAP evolution equations, can make sense if the accuracy of above calculation have the precision equal or higher than the

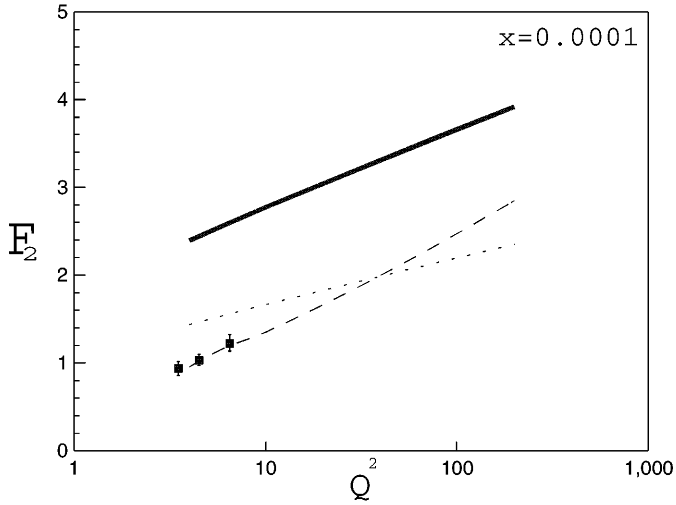

Fig. 1. The unpolarized proton structure function $F_2(x, Q^2)$ versus Q^2 (GeV^2) for fixed x=0.0001. The heavy, dashed and dotted curves are for different schemes according to text explaination of Sect. 4. The data are from [4]

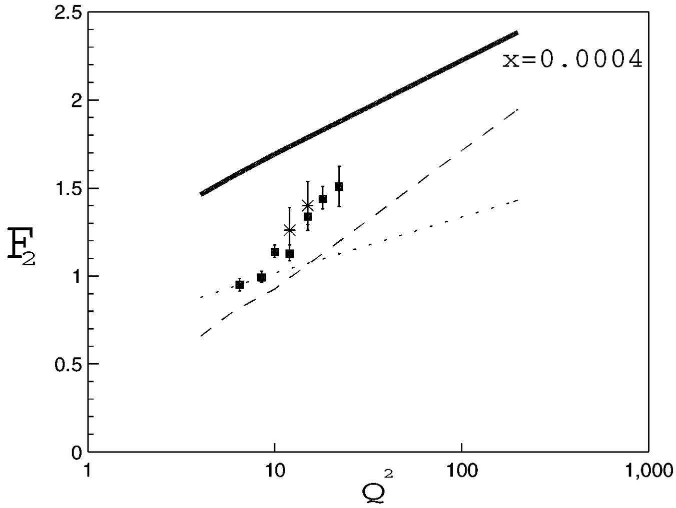

Fig. 2. As Fig. 1 but for x=0.0004

experimental data. However,in general, we have got a reasonable results compared to the available data.

In Figs. 7 to 14 we have plotted the ratio $A_1(x, Q^2) = g_1(x, Q^2)/F_1(x, Q^2)$ versus Q^2 for different value of x i.e., x= 0.014, 0.025, 0.035, 0.049, 0.077, 0.122, 0.173 and 0.25. The data have been taken from [6,7]. For these figures the polarized spin structure function have been calculated using Sect. 2 (in order to get polarized valence distributions at μ_0^2) and Sect. 3.2 (in order to find polarized valence and sea quark distributions at some Q^2 value) with three flavors. Then, (41) have been used to get the final result on $g_1(x, Q^2)$. $F_1(x, Q^2) = F_2(x, Q^2)/x$ has been taken from figures 1 to 6. Again it is seen that our results are reasonably close to the experimental data and for each x value the ratio has a very smooth behavior with respect to Q^2.

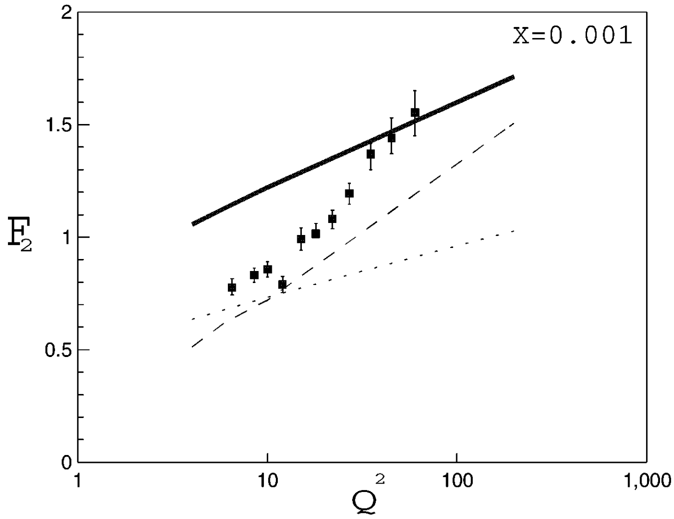

Fig. 3. As Fig. 1 but for x=0.001

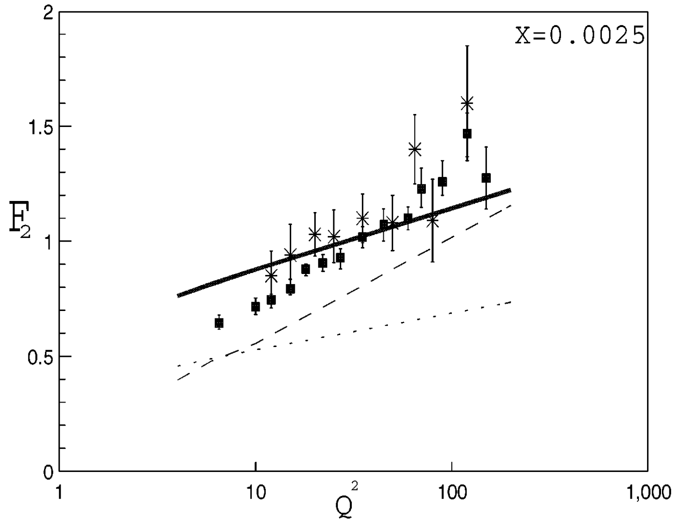

Fig. 4. As Fig. 1 but for x=0.0025

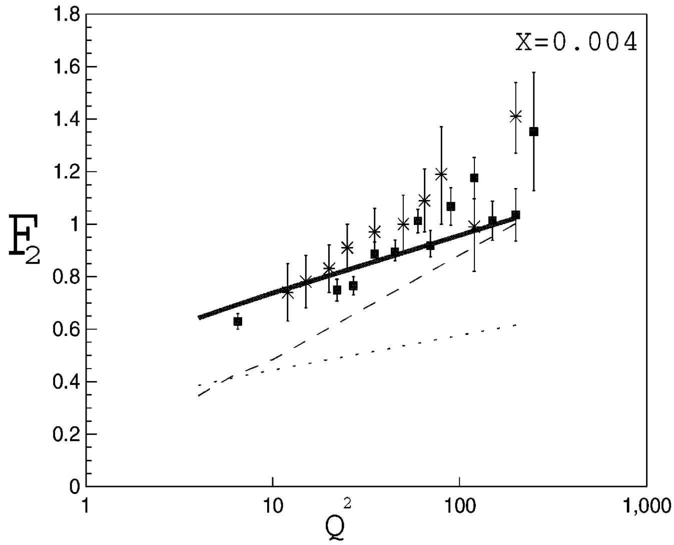

Fig. 5. As Fig. 1 but for x=0.004

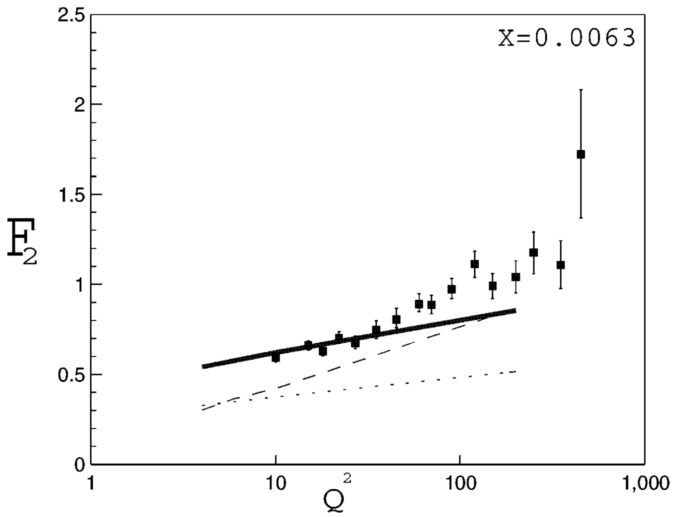

Fig. 6. As Fig. 1 but for x=0.0063

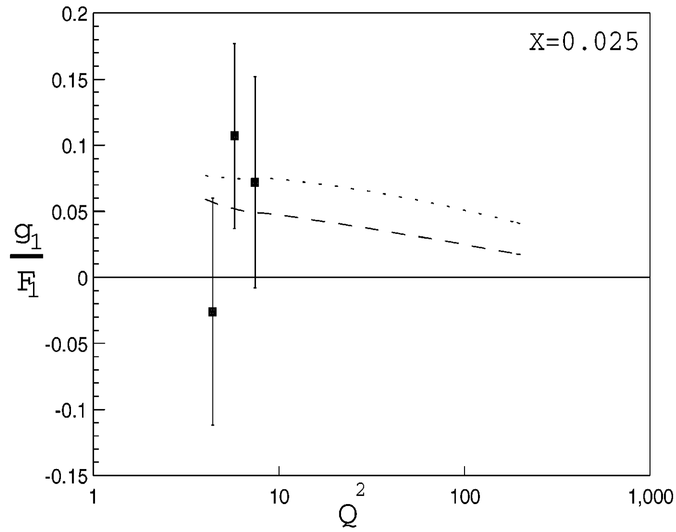

Fig. 8. As Fig. 7 but for x=0.025

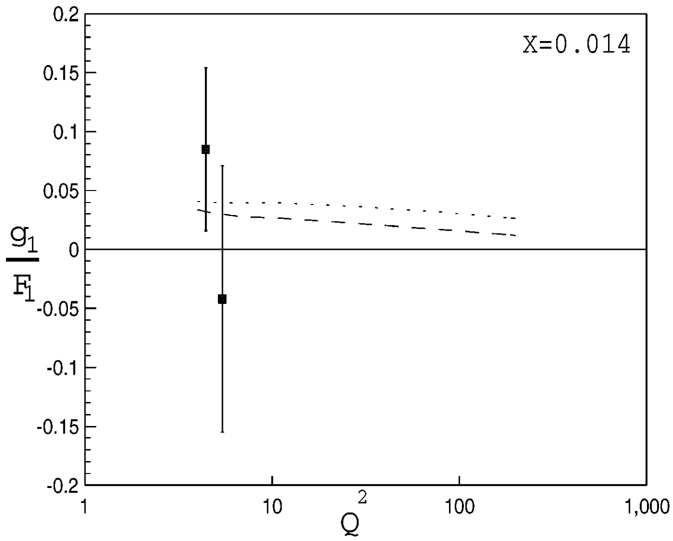

Fig. 7. The ratio of polarized to unpolarized proton structure function $A_1(x,Q^2) = g_1(x,Q^2)/F_1(x,Q^2)$ versus Q^2 at x=0.014. The dotted and dashed curves are the results of different schemes according to the text explanation of Sect. 5. The data are from [5] and [6]

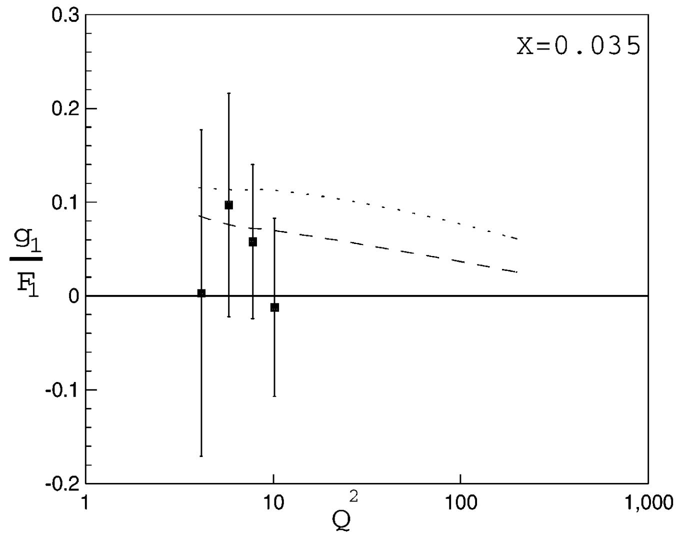

Fig. 9. As Fig. 7 but for x=0.035

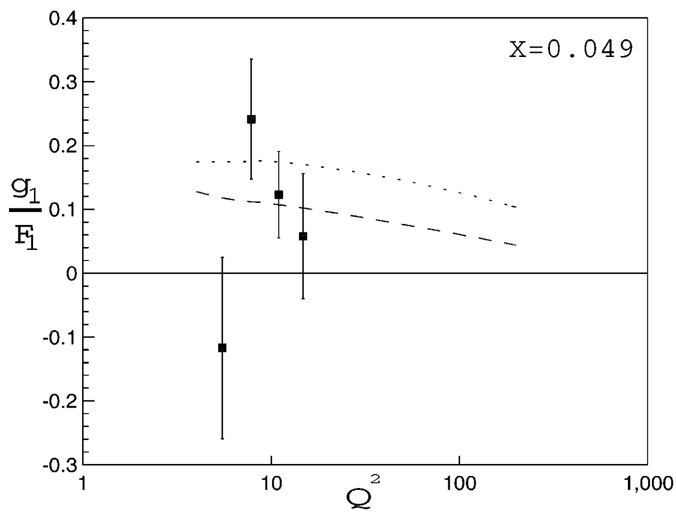

Fig. 10. As Fig. 7 but for x=0.049

In summary, we have found that the polarized and unpolarized nucleon structure function have approximately the same Q^2-dependence as the data and the whole results are consistent with the available experiments. Our calculation shows a similar scaling violation as the one observed in the experiment for the small x. The idea that at low x the scaling violation of $F_2(x,Q^2)$ comes from the gluon density alone and does not depend on the quarks densities was tested and it was shown that this approximation is only valid under certain conditions. The data can reasonably be explained by the evolution equation and the analytical approximation presented here. So in this respect we may conclude that the gluons are the dominant source of the partons in the small x region. However further work should be done in this direction.

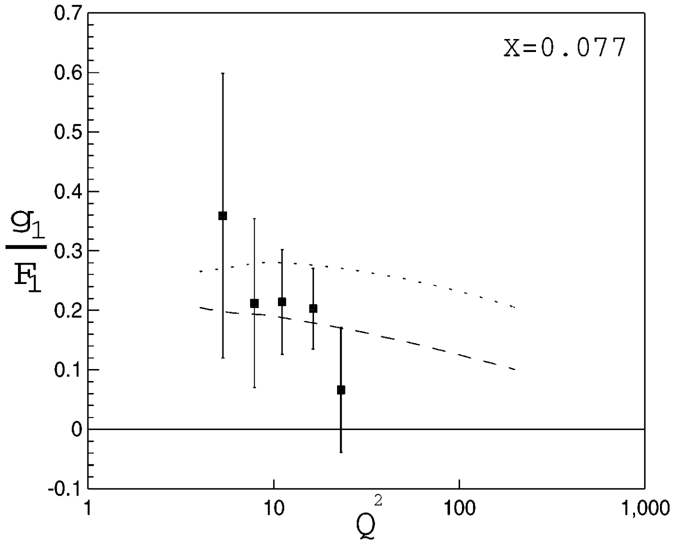

Fig. 11. As Fig. 7 but for x=0.077

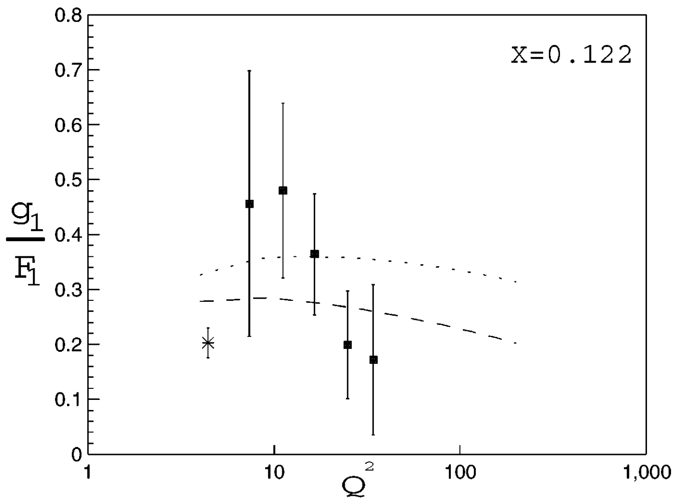

Fig. 12. As Fig. 7 but for x=0.122

In this work we have not considered the effect of next-to-leading order (NLO) corrections to the calculated parton distributions. In [16] we find that NLO corrections are not sizable for $Q^2 = 4\mathrm{GeV}^2$. On the other hand in [28] it has been shown that the NLO corrections are not very important for $x \geq 10^{-3}$. However we will investigate this matter in our future works in details.

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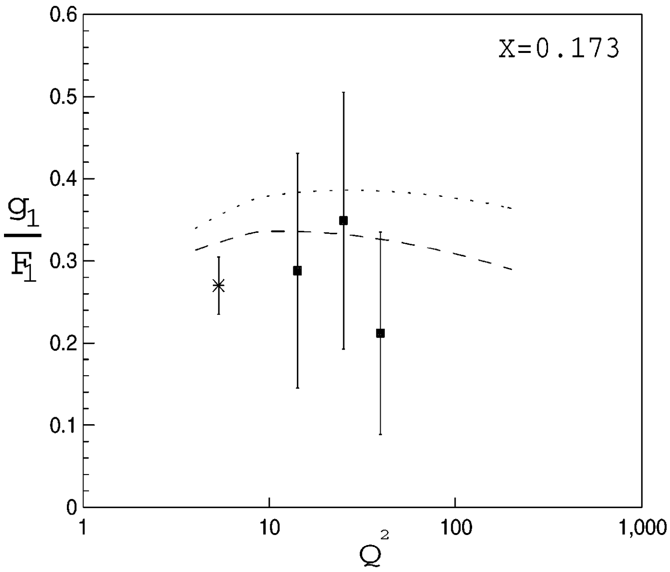

Fig. 13. As Fig. 7 but for x=0.177

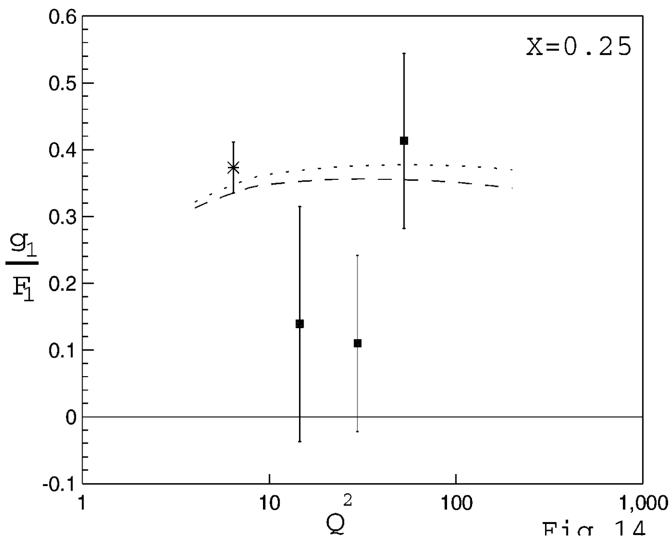

Fig. 14. As Fig. 7 but for x=0.25

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